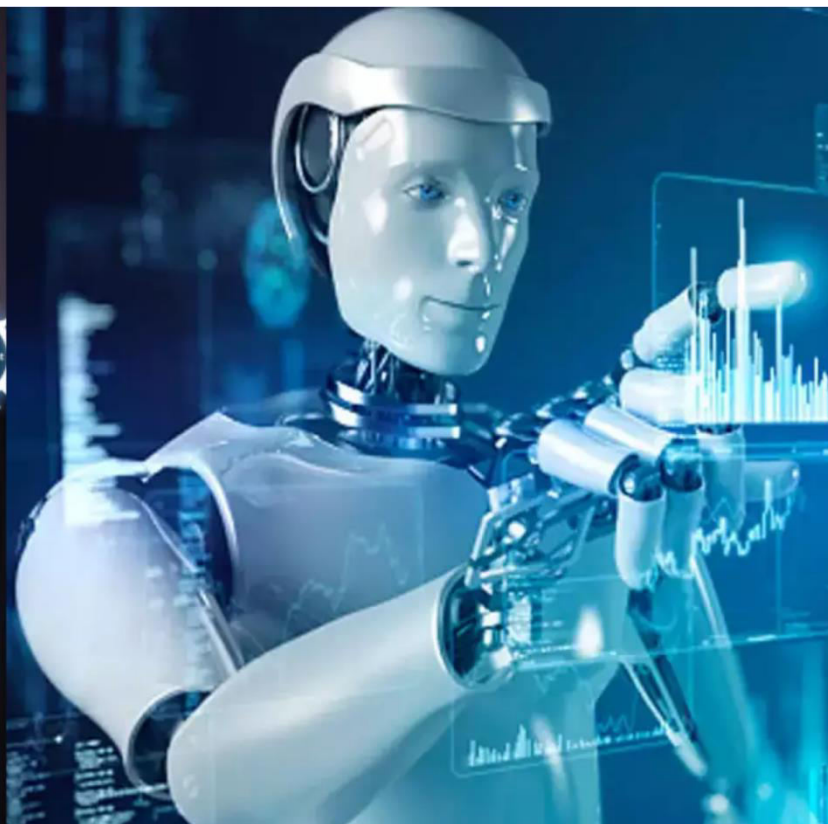




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Scholastic Approach of Bipotency in Near Rings

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ABSTRACT: The study of near-rings occupies a significant position within modern algebra due to their capacity to generalize ring structures and model non-associative or non-commutative algebraic systems. This paper investigates the structural properties and operational dynamics of near-rings under specific algebraic constraints. Emphasis is placed on the characterization of ideal structures, distributive identities, and homomorphic images that contribute to the internal symmetry and external algebraic correspondence of near-ring systems. The structure of Near ring is discussed with the left bipotency and its salient conditions. The role of S and S' near rings is induced with the left bipotency condition. Homomorphic structure of left bipotency is preserved. The research introduces a refined framework for analysing mappings and substructures, particularly focusing on the interplay between regularity and nilpotent elements that deviate from full distributivity.

KEYWORDS: Left Bipotent Near rings, S Near rings, Regular, Reduced Near ring

I. PRELIMINARIES

Definition 1.1 [4]

A (right) Near ring is a set N together with two binary operations “+” and “.” such that

- $(N, +)$ is a group (not necessarily abelian),
- $(N, .)$ is a semigroup
- $(n_1 + n_2) n_3 = n_1 n_3 + n_2 n_3$, for all n_1, n_2, n_3 in N (right distributive law)

Definition 1.2 [1]

The function $f: R \rightarrow R'$ is said to be a *homomorphism* if:

- $f(a+b) = f(a) + f(b)$
- $f(ab) = f(a) f(b)$, for all a, b in R

Definition 1.3 [2]

An element a in G is said to be an *idempotent* element if $a^2 = a$ and the set of all idempotents is denoted by E .

Definition 1.4 [4]

A near ring R is said to be *zero commutative* if $ab = 0$ implies $ba = 0$, for all a, b in R .

Definition 1.5 [8]

The function $f: R \rightarrow R'$ is said to be an *anti-homomorphism* if:

- $f(a+b) = f(b) + f(a)$
- $f(ab) = f(b) f(a)$, for all a, b in R

Definition 1.6 [4]

An element a in a near-ring is called *regular* if there exists x in N such that $a = axa$.

Definition 1.7 [9]

A near ring N is said to be *left bipotent* if $Na = Na^2$ for every a in N .

Notation 1.8 [4]

1. E denotes the set of all idempotents of N
2. L is the set of all nilpotent elements of N .
3. $C(N) = \{n \in N | nx = xn \text{ for all } x \in N\}$.

Definition 1.9 [7]

N is said to fulfil the Insertion of Factors Property (IFP) if $ab = 0 \Rightarrow anb = 0$ for all a, b, n in N . If in addition $ab = 0 \Rightarrow ba = 0$ for $a, b \in N$, we say N has $(*, \text{IFP})$

Definition 1.10 [2] A onto homomorphism is called an epimorphism. Remark 2.14 [4] If I is any ideal of N , then $\varphi : N \rightarrow N/I$ defined by $\varphi(x) = I + x$ for $x \in N$ is a near ring epimorphism and is called canonical homomorphism.

Remark 1.11 [9]

If I is any ideal of N , then $\varphi : N \rightarrow N/I$ defined by $\varphi(x) = I + x$ for $x \in N$ is a near ring epimorphism and is called canonical homomorphism.

Definition 1.12 [4]

A non-empty subset I of N is called

- (i) a left ideal of N if $(I, +)$ is a normal subgroup of $(N, +)$ and $n(n' + i) - nn' \in I$ for all $n, n' \in N$ and $i \in I$.
- (ii) (ii) a right ideal of N if $(I, +)$ is a normal subgroup of $(N, +)$ and if $IN \subseteq I$. (iii) an ideal of N if I is both a left ideal and a right ideal of N . Clearly $\{0\}$ and N are ideals of N . These are called trivial ideals.

II. MAIN RESULTS

Theorem 2.1

Every Left Bipotent near ring is an S near ring

Proof: Let N be a left bipotent near ring (i.e.) $Na = Na^2, \forall a \in N$

Then, $a \in Na^2 = Na$

$\Rightarrow a \in Na$

So, N is a S near ring which is to be proved.

Result 2.2

Any left bipotent sub-commutative near ring is an S' near ring

Proof: Let N be a left bipotent near ring (i.e.) $Na = Na^2, \forall a \in N$ -----(i)

N is sub-commutative, then $Na = aN, \forall a \in N$ -----(ii)

Let $a \in Na^2 = Na = aN$ (by (ii))

$\Rightarrow a \in aN, \forall a \in N$

Therefore, N is a S' near ring

Theorem 2.3

Homomorphic image of a left bipotent S' near ring are also so.

Proof: Let $f: N \rightarrow N'$ be a homomorphic near rings N onto N'

Let N be a left bipotent near ring (i.e.) $Na = Na^2, \forall a \in N$ -----(i)

If $a \in N'$, then there exists $b \in N$ such that $f(b) = a$

We know that, $Nb = Nb^2$

Then, $f(Nb) = f(N) f(b) = N'a$ -----(ii)

$f(Nb^2) = f(N)f(b^2) = f(N) [f(b)]^2 = N'a^2$

We know that, $Na = Na^2$

Therefore, $N'b = Nb^2$

Now, since $b \in bN$ we have, $a = f(b) \in f(bN) = aN'$

Theorem 2.4

A regular near ring N is left bipotent if each idempotent of N is central.

Proof: Let N be a regular near ring (i.e) $\forall a \in N$, there exists $x \in N$, such that $a = axa$ --(i)

N has idempotent e (say) and let $ax = e$.

To prove: N is a left bipotent near ring (i.e); $Na = Na^2$, $\forall a \in N$ -----(i)

Now, $(xa)^2 = (xa)(xa) = x(axa) = xa$ (by (i))

Now $a = a(xa) = (xa)a$ [since, each idempotent in N is central]

$\Rightarrow a = (xa)^2$ -----(ii)

To Prove: $Na^2 = Na$, $\forall a \in N$

Let $b \in Na$, then $b = na$, for some $n \in N$

$b = n((ax)a)$ (by (i)) $= n((xa)a) = (nx)a^2 \in Na^2$

$\Rightarrow Na \subseteq Na^2$

Let $d \in Na^2$, then $d = xa^2$ for some $x \in N$

$d = (xn'x)a^2$ (by (i)) for some $n' \in N$

$= xn'(xa^2) = (xn')a$ (by (ii))

$\in Na$

$\Rightarrow Na^2 \subseteq Na$

Therefore, $Na = Na^2 \forall a \in N$

So, N is a left bipotent near – ring

Theorem 2.5

Let N be a S near-ring, then N is regular if and only if a $(\neq 0)$ in N , there exists an idempotent e such that $Na = Ne$.

Proof: Let N be a S near ring

(i.e; $x \in Nx$, for every $x \in N$)

Since N is regular, for every $a \in N$, there exists $x \in N$, such that $a = axa$

Now to prove that for every $a (\neq 0)$ in N , there exists an idempotent e such that $Na = Ne$.

Let $xa = e$

Now $(xa)^2 = (xa)(xa) = x(axa) = xa$

$\Rightarrow e^2 = e$ which shows that e is idempotent

$Na = N(axa) \subseteq Nxa = Ne$

$\Rightarrow Na \subseteq Ne$ and $Ne \subseteq Na$ (obvious)

Therefore, $Na = Ne$

Conversely, given that N is a S near ring (i.e; $x \in Nx$, for every $x \in N$)

And, for every $a (\neq 0)$ in N , there exists an idempotent e such that $Na = Ne$.

Now to prove that N is regular, for every $a \in N$, there exists $x \in N$, such that $a = axa$

For any $d \in N$, there exists an idempotent b , such that, $d \in Nd = Nb$

$d = ub$ for some u in N

Also, $b \in Nb = Nd$

$b = yd$, for some y in N

Now, it is enough to prove that $dyd = d$

$dyd = ubyd = ubb = ub^2 = ub = d$

Therefore, N is a regular near ring

Theorem 2.6

A left bipotent near ring N is regular if and only if N is an S near ring

Proof: Let N be a left bipotent near ring (i.e; $Na = Na^2$, $\forall a \in N$ -----(i)

Given that N is regular, for every $a \in N$, there exists $x \in N$, such that $a = axa$ -----(ii)

Now to prove that N is a S near ring

Let $xa = e$

$(xa)^2 = (xa)(xa) = x(axa) = xa$ (by (i))

So, xa is idempotent

Now, $a = (ax)a = (xa)a$ (by theorem 2.3)
 $= xa^2 \in Na^2 = Na$ (by (ii))

$\Rightarrow a \in Na$ which implies that N is a S near ring

Conversely, let N be an S near ring and left bipotent near ring

Now to prove that N is regular, for every $a \in N$, there exists $x \in N$, such that $a = axa$

Let $a \in Na = Na^2$ (by (i))

$a \in Na^2$

Then $a^2 = za^4$, for some z in N

Then, $(a^2a^2 - za^4a^2) = 0$

$(a^2 - za^2a^2)a^2 = 0$ and $(a^2 - za^2a^2)za^2a^2 = 0$ (since N has I.F.P)

$(a^2 - za^2a^2)^2 = (a^2 - za^2a^2)(a^2 - za^2a^2) = (a^2 - za^2a^2)a^2(a^2 - za^2a^2)za^2a^2 = 0$

$\Rightarrow (a^2 - za^2a^2)^2 = 0$

$\Rightarrow a^2 = za^2a^2$ -----(iii)

Let $za^2 = e$

$(za^2)^2 = (za^2)(za^2) = z(a^2za^2) = za^2$ (by (iii))

$\Rightarrow za^2$ is idempotent

and $Na = Na^2 = Nza^2a^2$ (by (iii))

$\subseteq Nza^2 = Ne$

$\Rightarrow Na \subseteq Ne$ and $Ne \subseteq Na$ (obvious)

So, $Na = Ne$

Then N is regular near ring which follows from theorem 2.4

Theorem 2.7

A left bipotent near ring is an S near ring if and only if it is reduced

Proof: Given that N is a left bipotent near ring, $Na = Na^2$, for every a in N -----(i)

N is S near ring (i.e.); $x \in Nx$, for every x in N

Let $b \in N$ be nilpotent element (i.e) $b \neq 0$, $b \in N$

Now it is enough to prove that $b = 0$

For some positive integer n , $b^n = 0$

Then $b \in Nb = Nb^2 = \dots \dots \dots Nb^n$ and $b = 0$

Conversely, given that N is a left bipotent near ring

$L = \{0\}$

To Prove that N is a S near ring

If $x \in N$, Then $Nx = Nx^2$

So $x^2 = yx^2$ for some y in N

$x^2 - yx^2 = 0 \Rightarrow (x-yx)x = 0$

Also, $(x-yx)xy = 0$

$(x-yx)^2 = (x-yx)(x-yx) = (x-yx)x(x-yx)yx = 0$

Therefore, $x - yx = 0$

That is, $x \in Nx$, for every x in N

Therefore, N is S near- ring

Corollary 2.8

A left bipotent near ring is regular iff it is reduced

Proof: Follows by theorem 2.5 and theorem 2.6

Theorem 2.9

An S near ring is left bipotent iff $A = \sqrt{A}$ for every left N subgroup A of N

Proof: Given that N is a S near ring, that is, $x \in Nx$, $\forall x \in N$

N is left bipotent near ring, that is, $Na = Na^2 \forall a \in N$

Let A be a left N -subgroup of N

Now to prove $A = \sqrt{A}$

$A \subseteq \sqrt{A}$ (Obvious)

Let $a \in \sqrt{A}$, so it is enough to prove that $a \in A$

Then $a^n \in A$, for some integer n

We have $Na = Na^2 = \dots = Na^n$ is a left bipotent near ring

$a \in Na = Na^n$ (since, N is a left bipotent near ring)

$a = ban$, for some b in N

Thus, $a \in A \Rightarrow \sqrt{A} \subseteq A$

Therefore, $A = \sqrt{A}$

Conversely, given that N is a S' near ring and $A = \sqrt{A}$, where A is left N -subgroup of N

Let $a \in N \Rightarrow a^3 \in Na^2$ and $a \in \sqrt{Na^2} = Na^2$

Then $Na \subseteq Na^2 \subseteq Na \Rightarrow Na = Na^2$

Therefore, N is left-bipotent near ring

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